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Damage detection with spatial wavelets

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Abstract

This paper discusses a structural damage detection technique based on wavelet analysis of spatially distributed structural response measurements. The premise of the technique is that damage (e.g. cracks) in a structure will cause structural response perturbations at damage sites. Such local perturbations, although they may not be apparent from the measured total response data, are often discernible from component wavelets. The viability of this new technique is demonstrated with two examples: one based on numerically simulated deflection responses of a uniform beam containing a short transverse crack under both static and dynamic loading conditions, and the other based on smooth analytical crack-tip displacement fields. In each of these examples, the deflection or displacement response is analyzed with the wavelet transform, and the presence of the crack is detected by a sudden change in the spatial variation of the transformed response. This damage detection technique may serve the purpose of structural health monitoring in situations where spatially distributed measurements of structural response in regions of critical concern can be made with, for example, networks of distributed sensors, optical fibers, computer vision and area scanning techniques. It appears that this new technique does not require any analysis of the complete structure in question, nor any knowledge of the material properties and prior stress states of the structure. © 1999 Elsevier Science Ltd. All rights reserved.

1. Introduction

This paper presents a new method for structural damage detection and structural health monitoring. This method uses wavelet transforms to analyze spatially distributed signals (e.g. displacement and strain measurements) of a structure in regions of interest and detect damage by sensing local perturbations at damage sites. More classical methods, such as those based on frequency and stiffness analysis, extract damage information from variations in structural stiffness and natural frequency relative to a reference state (e.g. that of an undamaged structure). Based on simulation results obtained so far, it appears that this new method can provide an alternative to classical methods in damage detection and structural health monitoring, especially in situations

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where the classical methods are difficult to apply. Since the application of the wavelet transform to damage detection is a subject of on-going research, and further development is needed to make the new method practical, the purpose of this paper is to show the feasibility of the proposed method using simulated signals to shed light on the potential of the wavelet transform in analyzing spatially distributed structural response signals and in identifying local perturbations caused by damages such as cracks, and to stimulate further research in this area.

In the remainder of this section, we will first review some highlights of previous studies in damage detection (the review is by no means complete because of a large number of publications in this area), and then discuss briefly the current status of applications of the wavelet transform to vibration analysis and damage detection. In subsequent sections, we will describe concisely the wavelet theory and discuss how it will be used in this work, and we will demonstrate the viability of the new method through four simulation examples, three of which involve a beam containing a short transverse crack under both static and dynamic loading conditions and subject to different boundary constraints, and the last of which is based on a two-dimensional analytical crack-tip displacement response. At the end of the paper we will summarize the present study with conclusions and suggestions for future work.

1.1. Previous studies

Previous studies in damage detection have been focused on modal analysis of vibration signals in the time domain, which uses frequency and mode shape information. Early investigations were conducted by Schultz and Warwick (1971), DiBennedetto et al. (1972), and Adams et al. (Adams et al., 1975; Adams et al. 1978, Cawley and Adams, 1979a, 1979b). As pointed out in these studies, damage (e.g. cracks) in a structure results in a reduction in stiffness and an increase in damping, which in turn leads to a decrease in the natural frequencies of the structure. It was also argued that since the stress distribution through a vibrating structure is nonuniform and is different for each vibration mode, a localized damage would affect each mode differently, depending on the particular location of the damage. Based on the above premise, various damage detection methods have been devised and their feasibility demonstrated on laboratory test structures. The modal analysis based methodology proposed in the early works has also been closely followed and further investigated in many more recent studies, including those by Lee et al. (1987), Cawley and Ray (1988), Stubbs and Osegueda (1990), Saunders et al. (1992), and Armon et al. (1994), which mostly deal with damage detection in composite structures.

To detect damage using modal based methods, the vibration response of a structure before and after damage occurs is usually desired although a baseline is not always required (e.g. Stubbs and Kim, 1996). In addition, to locate and to quantify the damage a complete dynamic analysis of the structure is often performed and is usually carried out by solving a finite element representation of the structure. There are a number of inconveniences associated with the above requirements. First, it is not always possible or convenient to have the vibration response of a structure before it incurs damage (such as for aging aircraft). Second, it is not always feasible or practical to conduct dynamic analysis of a complete structure. Third, it is often difficult to obtain accurate material properties of a structure for a dynamic analysis. In addition, it has been argued by Banks et al. (1996) that it may be necessary to include the exact geometry of a damage in modal based damage detection methods, which is not easily done, and that the extension of modal based methods for simple structures (such as uniform beams) to more complicated structures may not be possible. It has been concluded by Banks and Inman (1991) that modal based methods are highly unreliable and inadequate for structures (e.g. composites) with variable material parameters.

To overcome some of the shortcomings of modal based methods, Banks et al. (1996) recently proposed an alternative method for damage detection with application in smart structures. This method is based on essentially the same premise as that used by modal based methods, namely that damage in a structure will correspond to changes in the structure's stiffness, damping and mass properties. However, unlike modal based methods, this alternative method does not extract damage information from frequency and mode shape variations. Rather, it directly utilizes variations in stiffness, damping ratio, and mass of a structure, as estimated from time histories of the input and vibration response of the structure, which are generated and measured by embedded and/or surface mounted piezoceramic patches. To determine the location and magnitude of damage in a structure, the response of the structure as measured by the piezoceramic patches are used to solve an inverse problem which is cast in this method as an optimization problem. This method has been experimentally validated for beam-like smart structures containing piezoceramic sensors and actuators. However, in order to use this method, the structure in question must be modeled mathematically by a proper boundary-value problem and solved iteratively using Galerkin approximation techniques. In light of this computational requirement, it appears that some of the limitations faced by modal based methods will also exist in this method.

If damage location is known in advance, such as at critical bolt joints, an electro-mechanical impedance method advanced by Rogers et al. (e.g. Liang, Sun and Rogers, 1996; Rogers and Giurgiatiu, 1997) has been shown to be very effective. In this method, an impedance probe consisting of a PZT exciter/sensor is attached to a potential damage site, which couples the internal impedance of the probe with that of the host structure in the immediate neighborhood of the probe. When damage occurs near the probe, the impedance output of the probe will change. Since this method is designed to detect localized damage at predetermined sites, it is not directly applicable to situations where damage may occur at arbitrary locations within an area.

1.2. Wavelet based methods

Although the literature on damage detection has been so far dominated by studies based on methods that utilize frequency or stiffness variation information, methods based on wavelet transform (e.g. Chui, 1992), a recently developed mathematical theory rooted in signal analysis (Daubechies, 1988, 1990; Mallat, 1989), are emerging. It is expected that a rapid growth in the application of wavelet transform to damage detection is poised to occur. It appears that Newland (1993, 1994a, 1994b) was the first to realize the potential of wavelet transform in vibrational signal analysis. The advantage of wavelet analysis, as opposed to Fourier or modal analysis, is that a wavelet transform breaks down a signal (for example, a time signal) into a series of local basis functions (wavelets) on the time axis and allows the identification of local features of a signal from the scale and position of the wavelets. Newland's introduction of wavelets to vibration analysis opens a new avenue of research although his work does not specifically address the issue of damage detection.

The first studies (that we are aware of) that use wavelets to perform damage detection analysis are those by Surace and Ruotolo (1994) and Wang and McFadden (1995, 1996). The first two authors used the wavelet transform to analyze simulated vibration response signal of a cracked beam as the crack in the beam opens and closes. On the other hand, the latter two authors applied the wavelet transform to actual gearbox vibration signals in the time domain to analyze the local features of these signals. Their work seems to indicate that gear damage can be correlated to features in time vs wavelet scale plots. However, further investigations are needed in order to demonstrate the effectiveness and reliability of this new method.

An application of wavelet theory in the space domain to crack identification in structures was recently proposed by Liew and Wang (1996). They suggested that wavelets in the space domain can be computed based on finite element or finite difference solutions of a mathematical representation of the structure in question under dynamic conditions. They argued that because the dynamic solutions contain information of the eigenfunctions (vibration modes) of the structure, the wavelets must be chosen to reflect the boundary conditions of the structure. As a numerical example, they considered the vibration of a simply supported uniform beam containing a transverse crack and solved the problem with a finite difference scheme. Then they calculated the wavelets along the length of the beam based on the numerical solution for the deflection of the beam. However, in order to locate the crack from the wavelet data, an initial displacement (excitation) that oscillates rapidly along the length of the beam was used to excite the beam. The crack location is then indicated by a peak in the variations of some of the wavelets along the length of the beam.

In the current paper, we propose that the wavelet transform be directly applied to spatially distributed structural response signals, such as surface profile, displacement, strain or acceleration measurements. It is conceivable that such measurements can be made over regions of critical concern with distributed networks of sensors, optical fibers, scanning methods, computer vision or other surface profile measurement techniques. We are interested in knowing whether local perturbations in the spatial signals at or near damage sites can be detected by wavelets due to the multiresolution property of the wavelet transform. We will present findings about the feasibility of this method based on simulated structural response signals. Our findings so far appear to show that both static and dynamic response measurements with sufficient accuracy can be used for damage detection. It also seems that the manner in which the structure is loaded has no particular significance, provided that the measurement data contain damage perturbation information. For most of the analysis, we will make used of the simple Haar wavelets (Haar, 1910). We will also demonstrate that other wavelets can also be used for this purpose (e.g. the Gabor wavelets).

2. Wavelet analysis

2.1. Wavelets and wavelet transform

Let $f(t)$ be a signal of interest in the time domain $(-\infty, \infty)$. Let $\Psi(t)$ be a complex-valued function localized in both time and frequency domains. We call $\Psi(t)$ the *mother wavelet*. Wavelets are generated from the mother wavelet by translation and dilation, as defined below

$$\Psi_{a,b}(t) = |a|^{-p}\Psi\left(\frac{t-b}{a}\right) \tag{1}$$

where a and b are real-valued parameters, and p is usually taken to be 1/2. The wavelet $\Psi_{a,b}$, associated with parameter a (dilation parameter) and b (translation parameter), oscillates at frequency a^{-1} and is positioned at time b. When a is very small, the interval in which the wavelet is non-zero contracts around the point b. The wavelet transform of the signal $f(t)$ is defined as

$$c_{a,b} = \int_{-\infty}^{\infty} f(t)\overline{\Psi_{a,b}(t)}\,\mathrm{d}t \tag{2}$$

where the overbar denotes the complex conjugate of the function under it. $c_{a,b}$ is called the *wavelet coefficient* for the wavelet $\Psi_{a,b}$. Wavelets with integer parameters are often used in wavelet transforms and, for example, can be generated from the mother wavelet according to

$$\Psi_{j,k}(t) = 2^{j/2}\Psi(2^j t - k) \tag{3}$$

where integers j and k are the dilation (scale) and translation (position) indices, respectively. The corresponding wavelet transform from eqn (2) can be denoted by $c_{j,k}$ (at scale j and position k).

The mother wavelet must justify the *admissibility condition*, given by

$$\int_{-\infty}^{\infty} |\Psi^*(\omega)|^2 \frac{\mathrm{d}\omega}{|\omega|} < \infty \tag{4}$$

where $\Psi^*(\omega)$ is the Fourier transform of $\Psi(t)$, that is

$$\Psi^*(\omega) = \int_{-\infty}^{\infty} \Psi(t)e^{-i\omega t}\,\mathrm{d}t \tag{5}$$

with $i = \sqrt{-1}$. A consequence of eqn (4) is that the mother wavelet must have a zero mean value.

The simplest wavelets were discovered by Haar (1910) long before the wavelet theory was formally established. The mother Haar wavelet is defined by

$$\Psi(t) = \begin{cases} 1, & 0 \leqslant t < 1/2 \\ -1, & 1/2 \leqslant t < 1 \\ 0, & \text{otherwise} \end{cases} \tag{6}$$

The mother Haar wavelet and several of its wavelets at different scales and positions are shown in Fig. 1 in the interval [0, 1].

Another family of wavelets used in this paper, the Gabor wavelets, is generated from the Gabor function below (e.g. Kishimoto, 1995):

$$\Psi(t) = \frac{1}{\sqrt[4]{\pi}}\sqrt{\frac{\omega_0}{\gamma}}\exp\left[-\frac{(\omega_0/\gamma)^2}{2}t^2 + i\omega_0 t\right] \tag{7}$$

where ω_0 and γ are positive constants. Following Kishimoto (1995), we choose $\omega_0 = 2\pi$ and $\gamma = \pi\sqrt{(2/\ln 2)}$. It is noted that the Gabor function approximately satisfies the admissibility condition (4) when γ is sufficiently large (e.g. the value taken in this study).

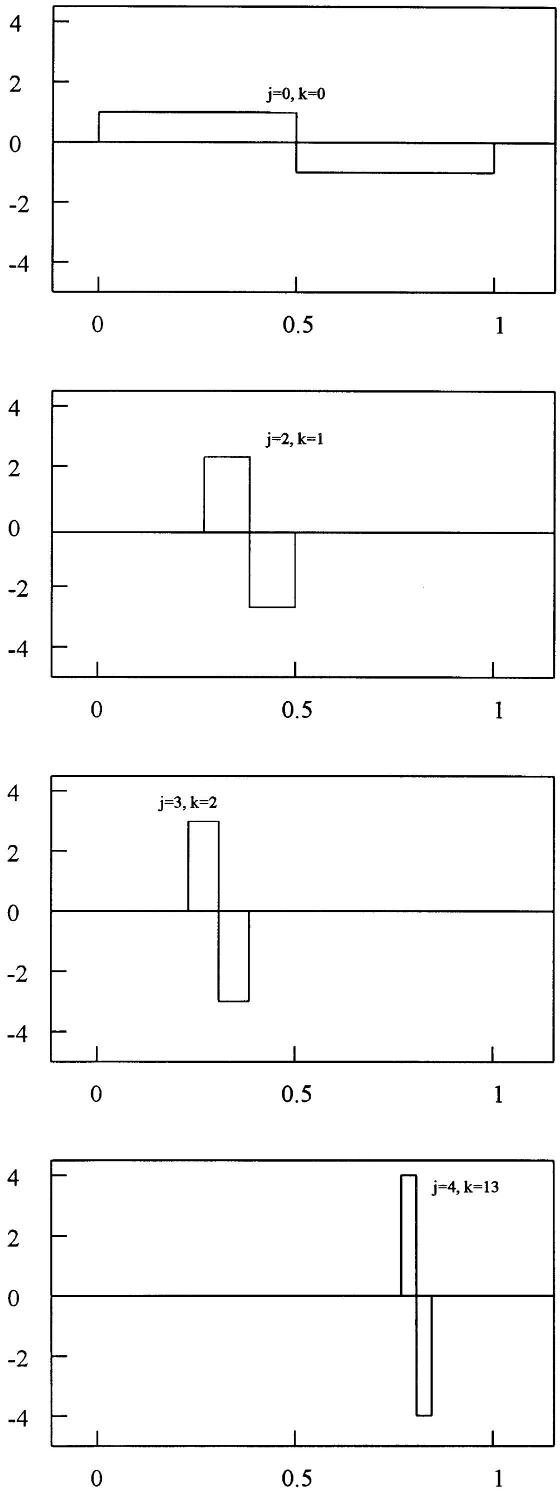


Fig. 1. Some examples of the Haar wavelets.

2.2. Damage detection by wavelet analysis of spatially distributed signals

Although wavelets are usually used to analyze signals in the time domain, spatially distributed signals can be equally analyzed with wavelets. We can do this by simply replacing time t with a spatial coordinate in previous equations. Without loss of generality, consider a spatial signal $f(x)$ distributed over [0, 1], where x refers to a spatial coordinate. For example, this signal can be a displacement or strain measurement over a region of interest for a structure under static or dynamic loading. Note that this region of interest (assuming it is one-dimensional for the purpose of this paper) can always be normalized to occupy [0, 1]. The wavelet transforms (wavelet coefficients) for the signal then can be determined with the integration limits running from 0 to 1, that is

$$c_{j,k} = \int_0^1 f(x)\overline{\Psi_{j,k}(x)}\,\mathrm{d}x \tag{8}$$

where j is the scale index and k the position index and the wavelets are defined by eqn (3).

Because of the wavelet transforms $c_{j,k}$ are performed with various scaled versions of the mother wavelet function, local perturbations in a signal $f(x)$ will be reflected in the fine-scale wavelets (those with relatively large j values) that are positioned (as indicated by the k value) at the locations of the perturbations. This is the *multi-resolution property* of the wavelet transform. In other words, local perturbations can be determined by performing the wavelet transform to obtain wavelet coefficients, and by examining the variations of the wavelet coefficients with position. For signals distributed in two dimensions, the two-dimensional wavelet transform can be used.

In light of the above discussion, we argue that wavelet transform can be used for structural damage detection as long as the spatially distributed signal contain local perturbations caused by the presence of damage (e.g. cracks) over the region where the signal is collected. Such signals may be obtained from surface or internal measurements of displacement, strain, or other quantities whose values can be disturbed by the presence of damage. A number of measurement techniques perhaps can be used for this purpose, such as surface mounted or imbedded networks of sensors, fiber optics, computer vision, and scanning techniques.

The procedure for this damage detection method is as follows. First, collect spatially distributed signals over an area of interest. Second, perform the wavelet transform to the signal to obtain wavelet coefficients for some *fine-scale* wavelets (e.g. wavelets of scales or levels $j = 8$ or higher). The reason why fine-scale wavelets must be used is that only wavelets of sufficient levels will be able to detect local variation in a signal. Third, for each level of wavelets, plot the value of the wavelet coefficients in the region spanned by the corresponding wavelets. Fourth, examine the distributions of the wavelet coefficients at each level. A sudden change (e.g. a peak) in the distribution of the wavelet coefficients signifies a strong local perturbation in the signal in the region spanned by the corresponding wavelet. If a detected perturbation is not caused by a known source (such as a known geometric or material discontinuity), then it will be attributed to the presence of damage at or near the side of the perturbation.

3. Feasibility demonstration

In this section we will present evidence about the feasibility of the wavelet-based damage detection method with simulated structural response data. The simulated signals will be analyzed

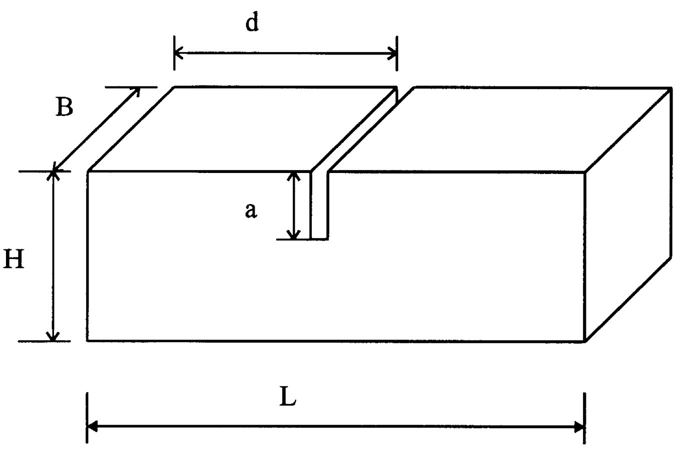


Fig. 2. A schematic of a uniform beam containing a transverse crack.

with the wavelet transform and the transformed signals will be studied to see if local features can be observed and identified near damage sites. A sensitivity analysis will also be performed.

We analyze two problems here. First, we consider the deflection response of a uniform beam containing a short transverse crack subjected to static or impact loading conditions, and then consider the displacement response of a large sheet structure (a plate under plane strain or plane stress deformation) containing a through-thickness crack. The deflection of the beam will be obtained numerically using a finite difference scheme. The displacement response of the cracked plate will be based on the analytical crack-tip field solution for an elastic body. These deflection or displacement response solutions are the simulation data here that serve the purpose of providing structural response signals when experimental measurements are not available.

For the beam problem, we consider a beam of length L, height H, and width B, as shown in Fig. 2. Let x be the coordinate along the length of the beam with its origin at the left end of the beam. Suppose the beam contains a transverse, through-thickness edge crack of length a at a distance of d away from the left end of the beam. For simplicity, we let the beam be made of a homogeneous and isotropic material with a Young's modulus E. The governing equation for the beam according to the Euler-Bernoulli beam theory is given by

$$EI\frac{\partial^4 w}{\partial x^4}+\rho A\frac{\partial^2 w}{\partial t^2}=q \qquad (0<x<L; t>0) \tag{9}$$

where w is the lateral deflection of the neutral axis of the beam, I the area moment of inertia of the beam's cross section, ρ the mass density, A the cross sectional area, t the time, and q the distributed transverse loading on the beam, which can be a function of x. When a crack is located at $x = d$ ($d < L$), the governing equation will be over two adjacent regions, (0, d) and (d, L).

In the finite difference scheme used here, eqn (9) will be discretized in the space domain with standard finite difference approximations on either side of the crack. The effect of the crack will be represented by a discontinuity in the rotation of the beam at the crack location, where the deflection, bending moment and shear force are held continuous. The jump in the beam's rotation across the crack location is approximated (see, Krawczuk and Ostachowicz, 1995) by the product of the beam curvature there, $\partial^2 w/\partial x^2$, and the length of the crack, a. (In order to show that the crack can be located without the embedded jump in rotation, we will later use a smooth displacement response along lines at a certain distance away from the crack tip and demonstrate that the crack tip can also be identified.) The resulting second-order ordinary differential equations with respect to time will be solved with a standard eigen-function decomposition procedure.

In the following, we will first consider the case of a simply supported beam subjected to a static point load. We will then analyze two more cases involving a cantilever beam subjected to an initial impact loading. In the first impact loading case, an impact load (treated as a delta function) is applied at the free end of the beam, which is to the right of the crack location, and in the second case, an impact load (also treated as a delta function) is applied at a mid point to the left of the crack location. Without attaching any particular significance to the parameter values chosen here, we let $L = 0.5$ m, $H = 2.5$ cm, $a = 0.25$ cm, $d = 0.252$ m, $B = 1.5$ cm, and use typical steel properties, $E = 210$ GPa and $\rho = 7.87$ kg/m^3, in all three examples. In each case, the resulting deflection distribution will be analyzed with Haar wavelets.

3.1. A simply supported beam under static loading

With this example we will show that damage detection can be accomplished with a static response of a damaged structure. This capability is important in that it opens up the possibility of detecting sub-surface damage (e.g. fatigue cracks and delaminations) by analyzing the surface profile of a statically loaded structure, which may be obtained by full-field measurement techniques (such as those based on computer vision) that are more suited for slowly varying signals.

The simply supported beam in Fig. 3 is loaded by a concentrated static force ($F = 100$ N) at a distance of one fourth of the beam length from the left end. The deflection, w, of the beam is shown

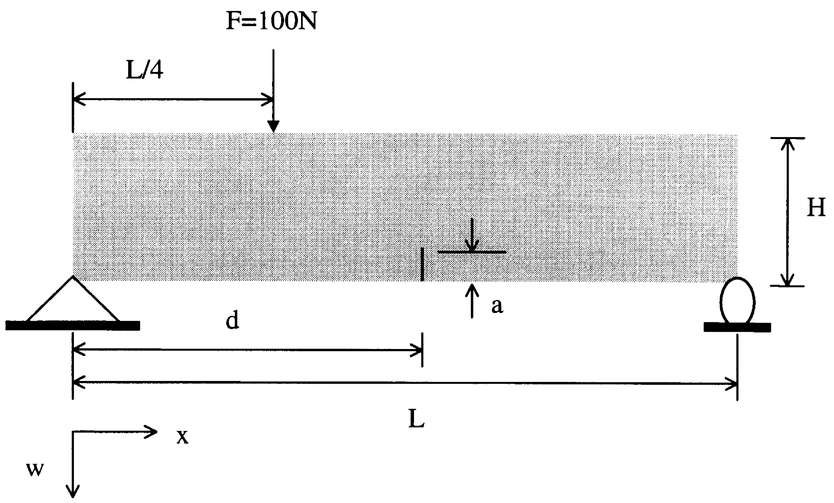


Fig. 3. A simply supported beam with a crack and under a static load.

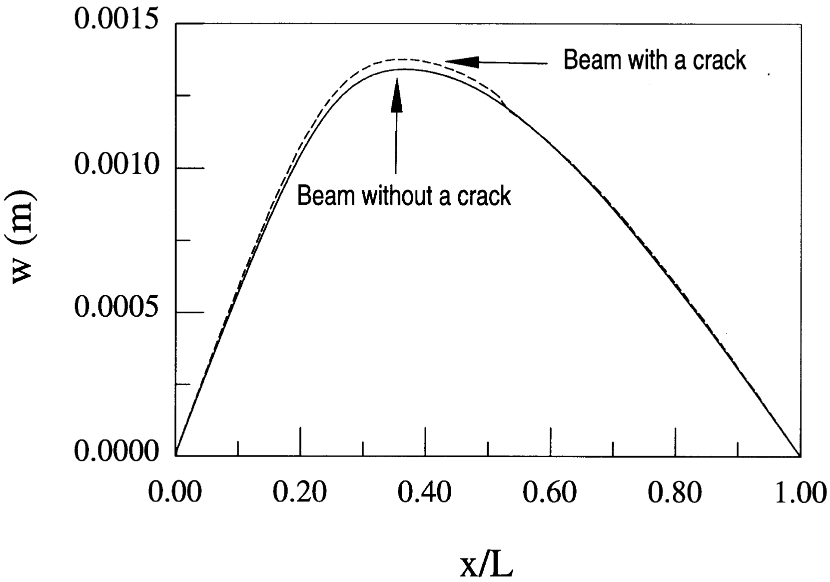


Fig. 4. The deflection response of the beam in Fig. 3 with and without the crack.

in Fig. 4 against the beam coordinate x normalized by the length of the beam L. For comparison, the response of the beam without the crack is also shown, although it is not needed in the present analysis.

Now we will transform the spatially distributed deflection signal for the cracked beam with Haar wavelets according to the procedure outline previously. The wavelet coefficients are obtained from eqn (8) for wavelet scales 1–10. The values of the coefficients are then plotted over the regions spanned by the corresponding wavelets along the x-axis, as shown in Fig. 5, where the numbers just outside the plots are the scales corresponding to the wavelet transforms. It is observed that while the first few wavelets (those of scales 7 and below) are not fine enough to detect local perturbations induced by the crack, wavelets of scales 8 and higher are able to identify the local perturbation by showing a peak at the position of the crack.

3.2. A cantilever beam under impact loading at the free end

This example demonstrates that a spatially distributed dynamic signal can also be used to detect damage using the proposed method. We consider a cracked cantilever beam whose left end is fixed and whose right end is subjected to an impact point load at time $t = 0$ (treated as a delta function with its magnitude, $F = 100$ N). The dynamic response of the cracked beam at 0.5 s after the impact is given in Fig. 6. Overall, the deflection of the beam is smooth except near the position of the crack where a small disturbance is observed. (Since the signal in Fig. 6 is a snap shot of the spatially distributed deflection variation, it is not obvious that it contains all the higher-order oscillation modes often associated with a delta function excitation. Had we presented the signal in the time domain for a fixed spatial point, we would have seen these higher-order modes.) When the deflection response signal is analyzed with the wavelet transform, this small disturbance is

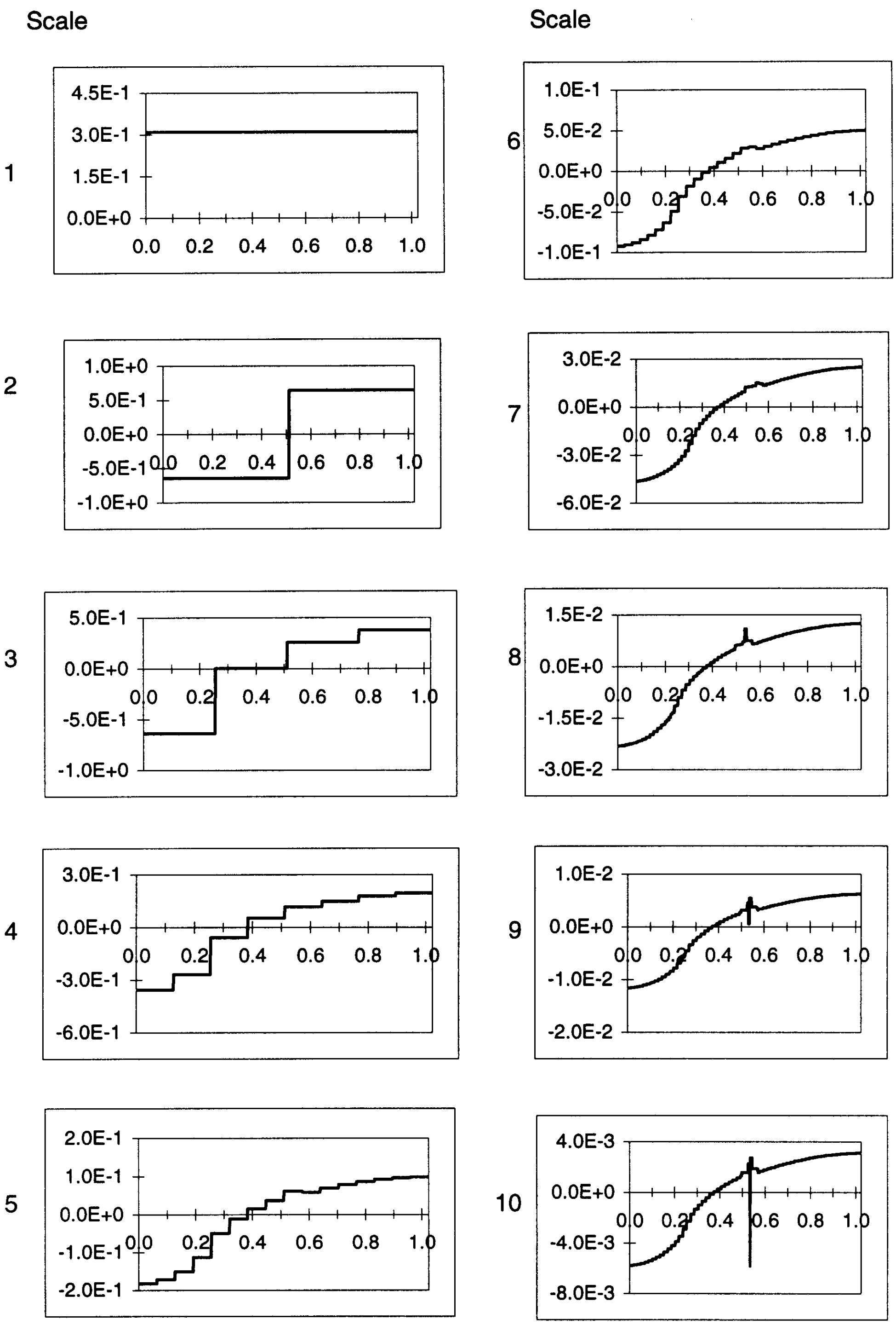


Fig. 5. The distributions of Haar wavelet coefficients for wavelets of scales 1–10 based on the deflection response in Fig. 4 of the cracked beam.

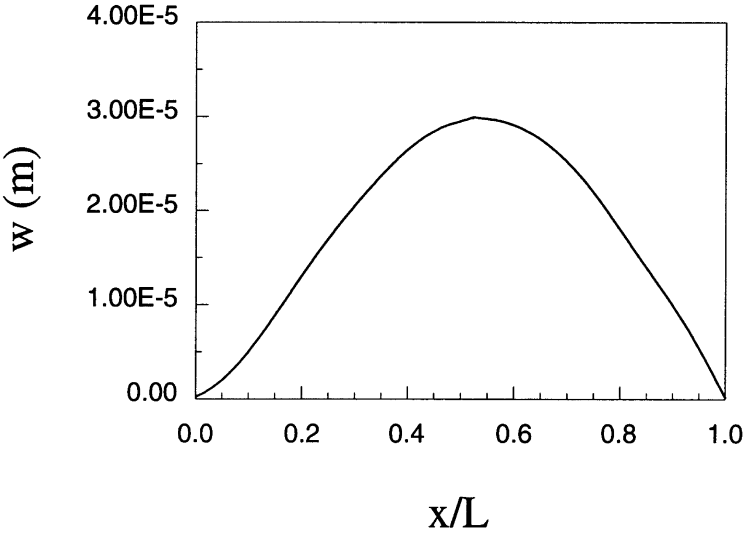


Fig. 6. The deflection response of the cracked cantilever beam loaded at the free end at $t = 0.05$ s after impact.

detected by wavelets of scales 8 and above. As shown in Fig. 7, rapid variations (local peaks) appear in the distributions of the Haar wavelet coefficients at the crack location.

3.3. A cantilever beam under impact loading at a mid point

In order to see whether the manner in which a cracked structure is loaded affects the ability of the wavelet-based method to detect damage (assuming the response signal contains damage-induced perturbations), we reconsider the preceding example with the position of the impact load moved from the free end to a mid point between the fixed end and the crack, at a distance of $L/3$ to the fixed end of the beam. The impact load is applied at $t = 0$ and is again treated as a delta function with a magnitude of 100 N. The spatially distributed deflection signal at $t = 0.05$ s after the impact is considered. Figure 8 shows the deflection variation with a small perturbation at the crack location. (Again, had we shown the signal in the time domain, we would have observed the usual higher-order modes often associated with a delta function excitation.) When this signal is analyzed with the wavelet transform, local peaks are observed at the crack position in the wavelet coefficient distributions for Haar wavelets of scales 6 and higher (see Fig. 9).

3.4. A large plate with a through-thickness crack

In the preceding examples, the effect of damage on the response of a structure is simulated in a one-dimensional setting via the simple beam theory. A question that may arise is whether the detection of damage by the spatial wavelets is a result of the particular one-dimensional approximation used in the simulations or it is a result of some inherent local feature induced by the damage. This question can be answered by the following example.

We consider a large plate containing a through-thickness crack of length $2a$ and subjected to a

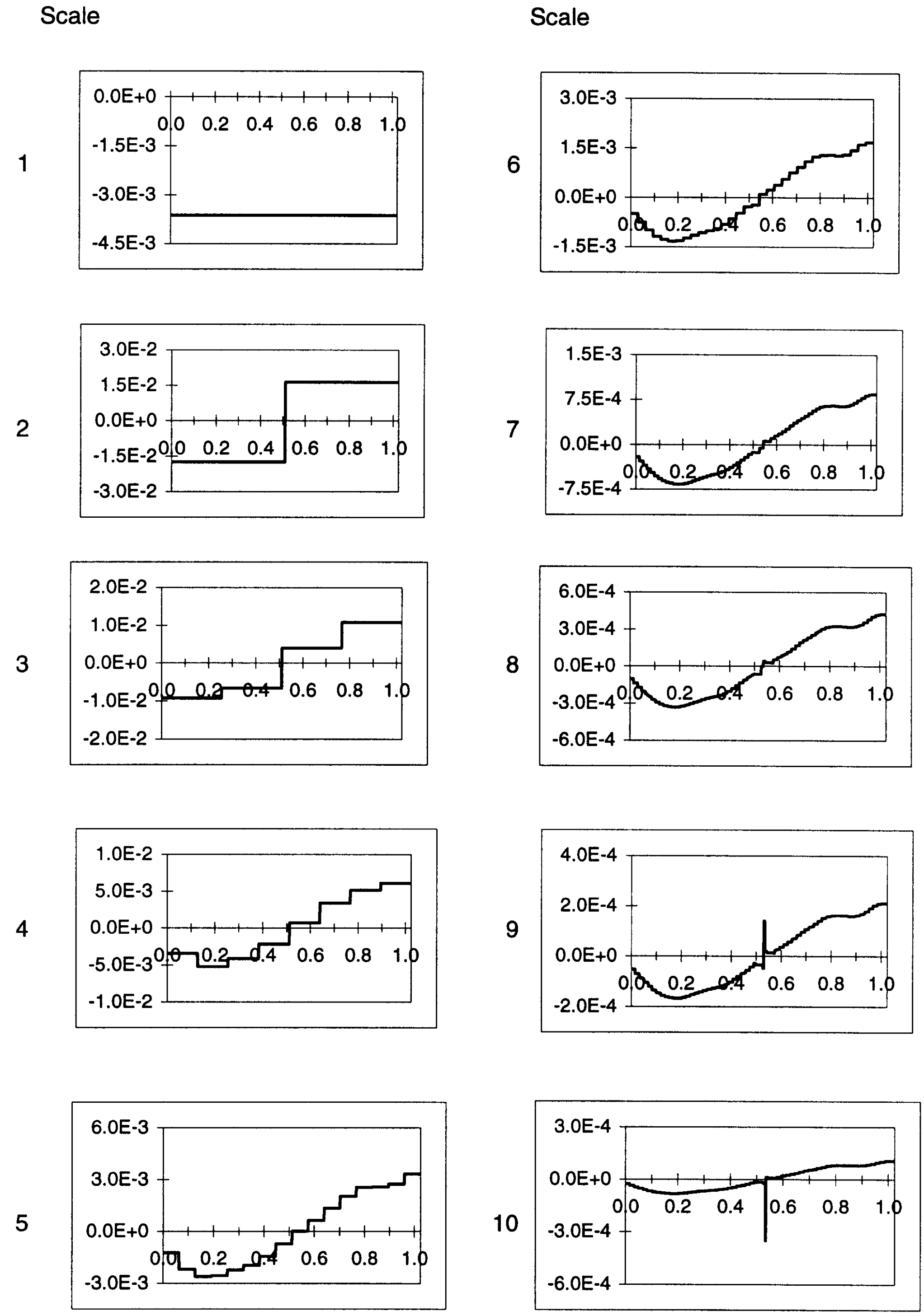


Fig. 7. The distributions of Haar wavelet coefficients for wavelets of scales 1–10 based on the deflection response in Fig. 6 of the crack beam.

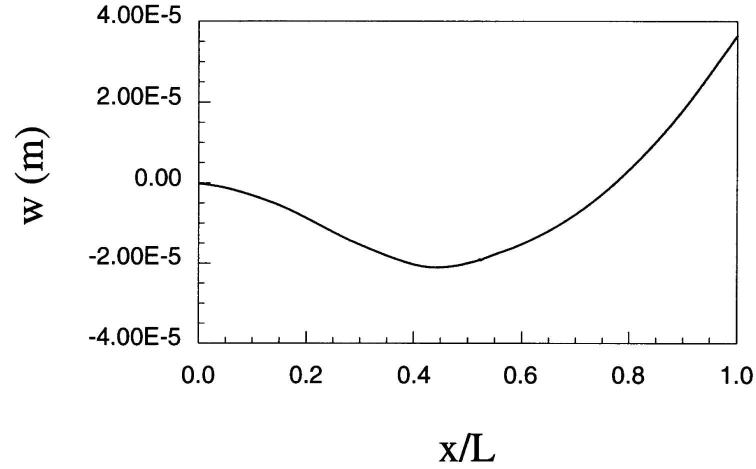


Fig. 8. The deflection response of the cracked cantilever beam loaded at a mid point at $t = 0.05$ s after impact.

remote uniform tension loading, as shown in Fig. 10(a). Under two-dimensional conditions (plane strain or plane stress) and assuming linear elastic behavior for the plate, the stress and deformation fields near the crack tip can be obtained analytically and are well-known in fracture mechanics. For example, the Cartesian rectangular displacement components of the near-tip displacement field can be written as

$$u_x = \frac{K_{\mathrm{I}}}{2G}\sqrt{\frac{r}{2\pi}}\cos\frac{\theta}{2}\left[k-1+2\sin^2\left(\frac{\theta}{2}\right)\right],$$

$$u_y = \frac{K_{\mathrm{I}}}{2G}\sqrt{\frac{r}{2\pi}}\sin\frac{\theta}{2}\left[k+1-2\cos^2\left(\frac{\theta}{2}\right)\right], \tag{10}$$

where (r, θ) are the crack-tip polar coordinates; G is the elastic shear modulus; $k = (3–4\nu)$ in plane strain and $(3–\nu)/(1+\nu)$ in plane stress, ν being the Poisson's ratio; and K_{I} is the stress intensity factor, which relates to the half crack length a and the remote tensile stress σ by $K_{\mathrm{I}} = \sigma\sqrt{(\pi a)}$.

Based on the displacement components in (10) we can generate displacement response data of the plate along lines at a certain distance away from the crack tip. For example, we can do this along a vertical line at $x = d$ or along a horizontal line at $y = h$, as shown in Fig. 10(b). Considering that (10) is valid only near the crack tip, we will take d and h to be $0.1a$ and we will generate u_x and u_y values in the interval of $-0.5a < y < 0.5a$ for the vertical line and in the interval of $-0.5a < x < 0.5a$ for the horizontal line. Plane stress conditions and common material properties for steels are used in the following discussions. Since all length dimensions will be expressed as a fraction of the half crack length, a, we have chosen a to be 1 cm for the sake of simplicity.

The displacement response along the vertical line segment, $x = 0.1a$ and $-0.5a < y < 0.5a$, is shown in Fig. 11(a), where the crack tip is closest to the mid-point ($y = 0$) of the line segment. It is clear that, while u_x component shows a dip at the crack-tip location, the u_y component reveals no local features that directly tell us where the crack tip is. Now the wavelet transform is applied

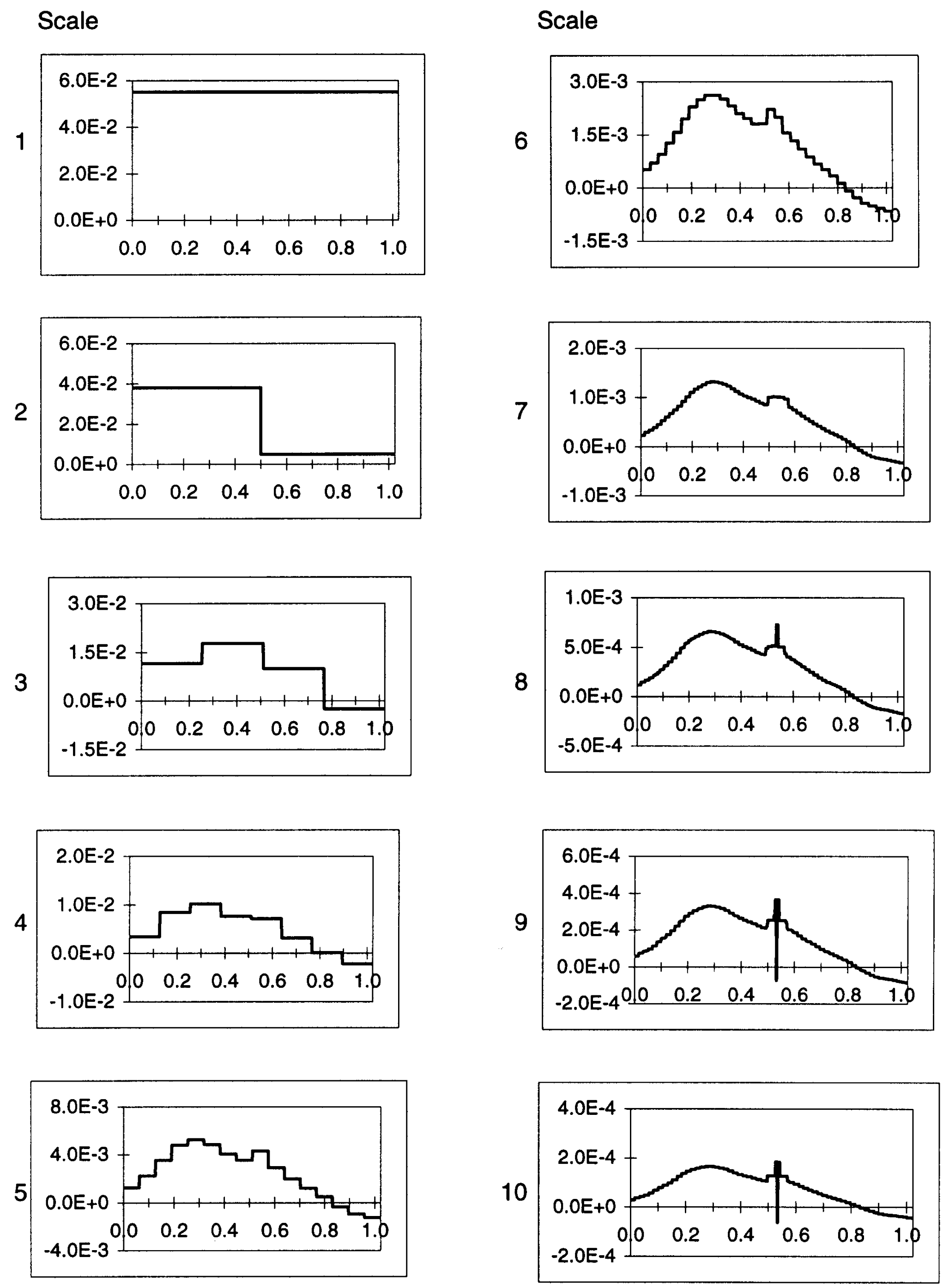
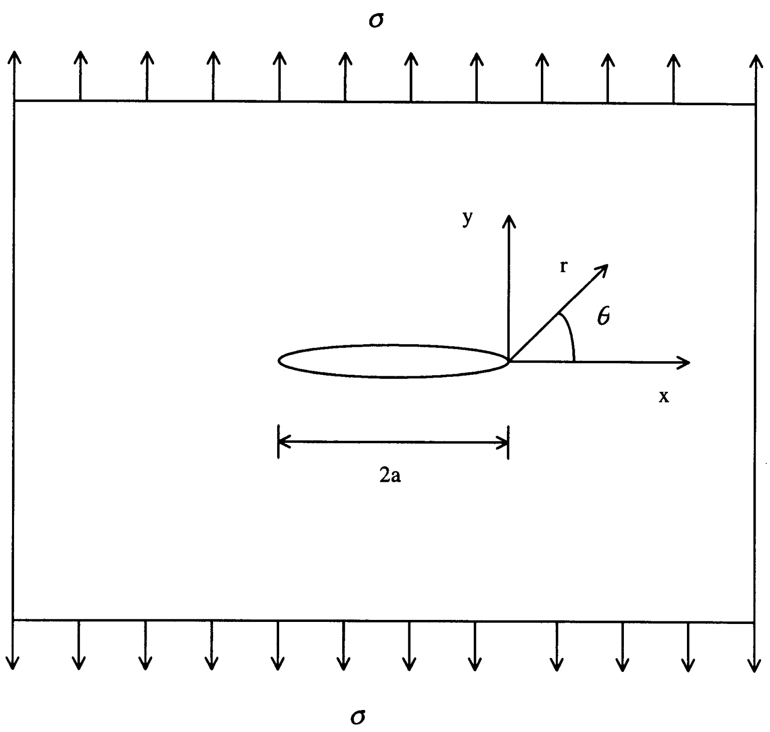
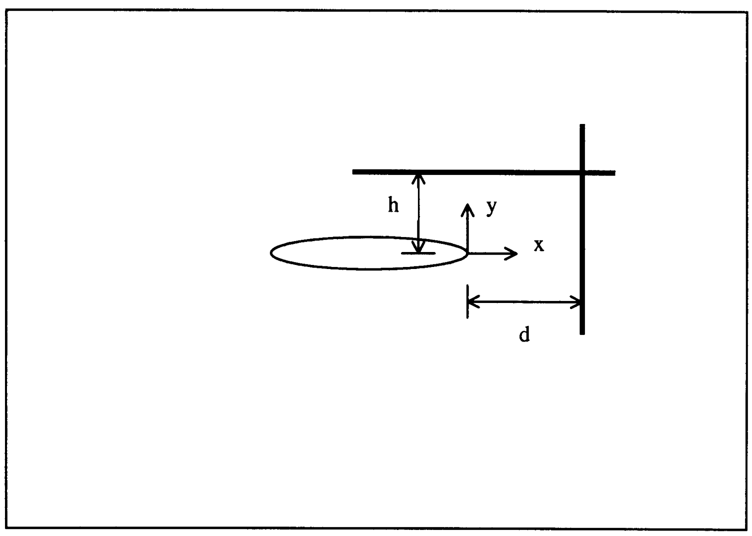


Fig. 9. The distributions of Haar wavelet coefficients for wavelets of scales 1–10 based on the deflection response in Fig. 8 of the crack beam.



(a)



(b)

Fig. 10. (a) An infinite plate containing a through-thickness crack and subjected to a remote uniform tensile loading; (b) a vertical and a horizontal line segments.

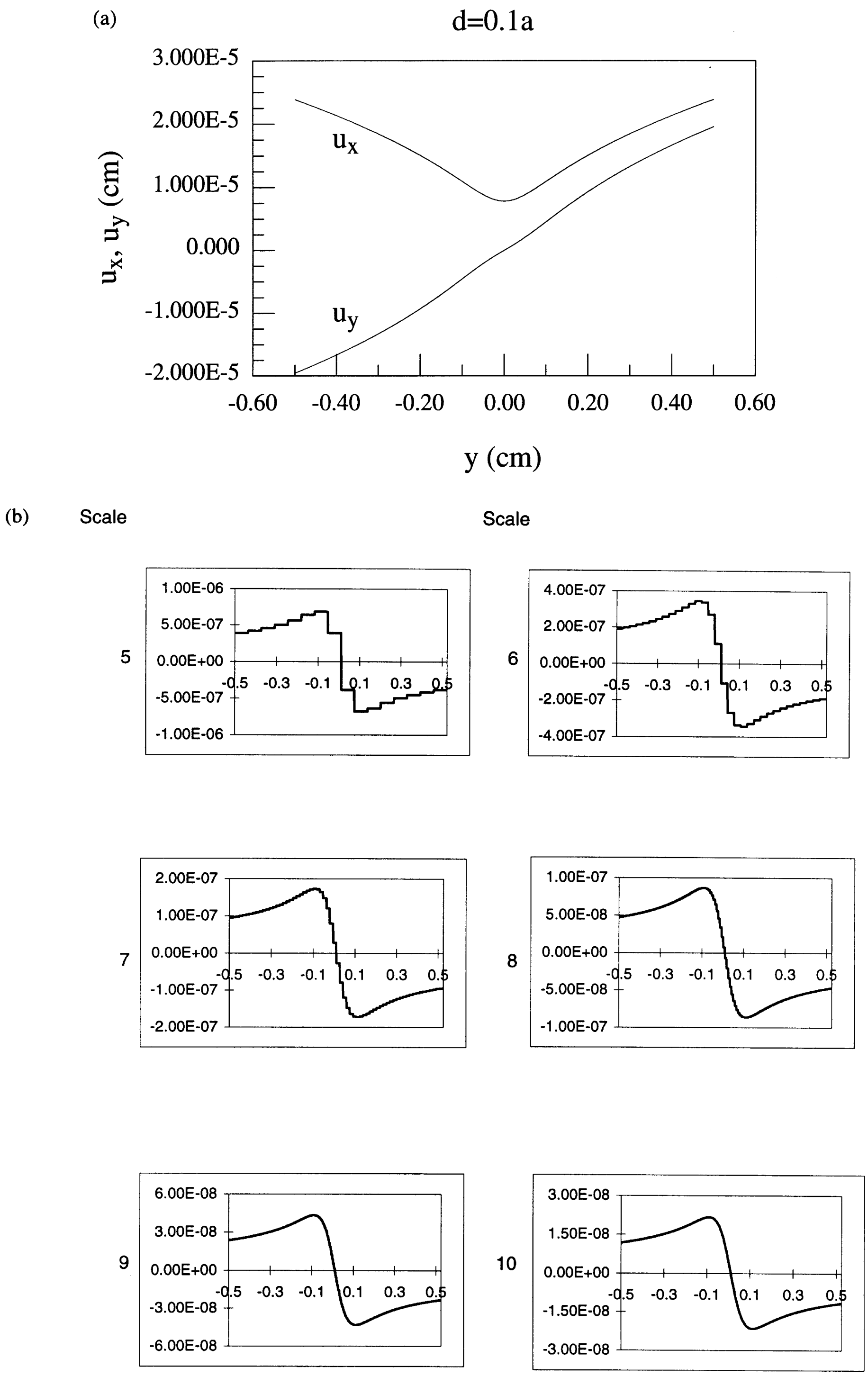


Fig. 11. (a) Displacement response along the vertical line segment; (b) Haar wavelets of scales 5–10 for u_x; (c) Haar wavelets of scales 5–10 for u_y.

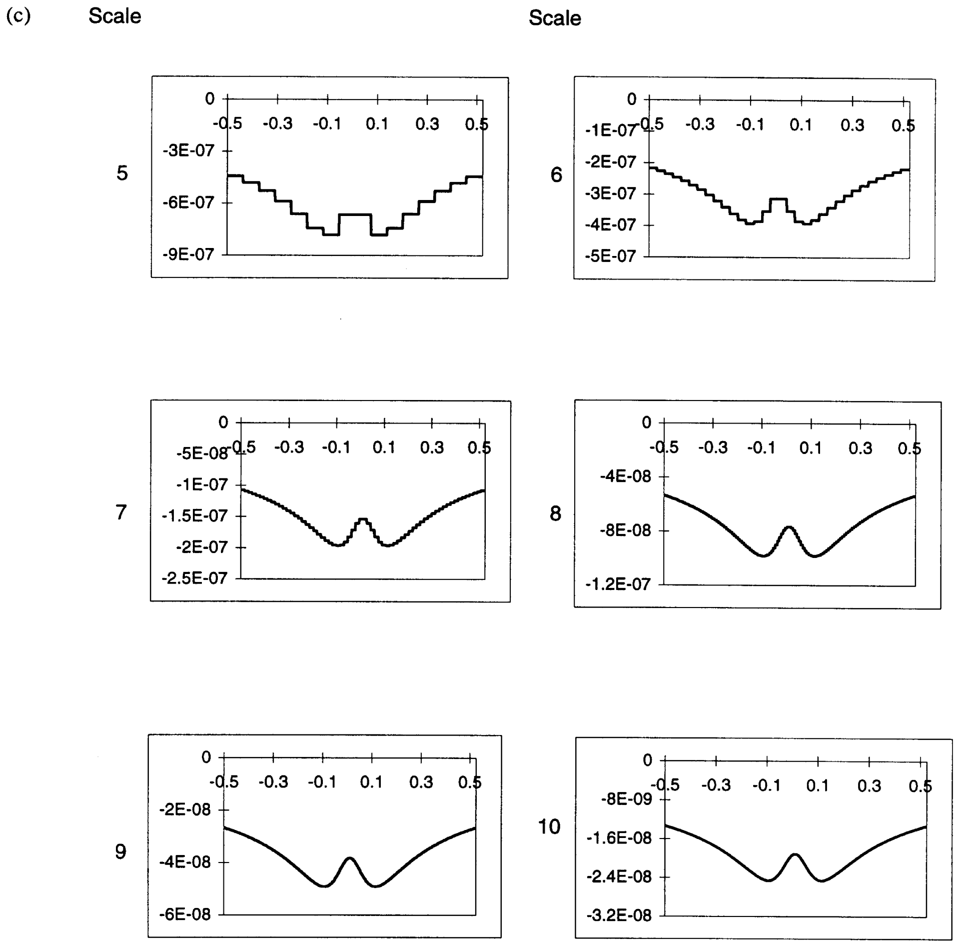


Fig. 11 continued.

to both displacement response distributions, and the resulting wavelet coefficient values of scales 5–10 are plotted against their positions along the line segment in Fig. 11, where Fig. 11(b) is for u_x and Fig. 11(c) is for u_y. In both figures, a sudden change is observed, indicating the crack tip location.

A similar observation can be made for the displacement response along the horizontal line segment, $y = 0.1a$ and $-0.5a < x < 0.5a$. From the displacement response shown in Fig. 12(a), it is very difficult to conclude that the crack tip is closest to the mid-point ($x = 0$) of the line segment. However, it is an easy matter to see from the wavelets of scales 5–10 (Fig. 12(b) for u_x and Fig. 12(c) for u_y) that the crack tip is located at the mid-point, where the strongest disturbance is observed.

In conclusion, this example demonstrates that structural damage, such as cracks, induce certain inherent perturbation features in the total structural response signals at locations nearest to the damage sites. It appears that these local features can be revealed by interrogating the spatial

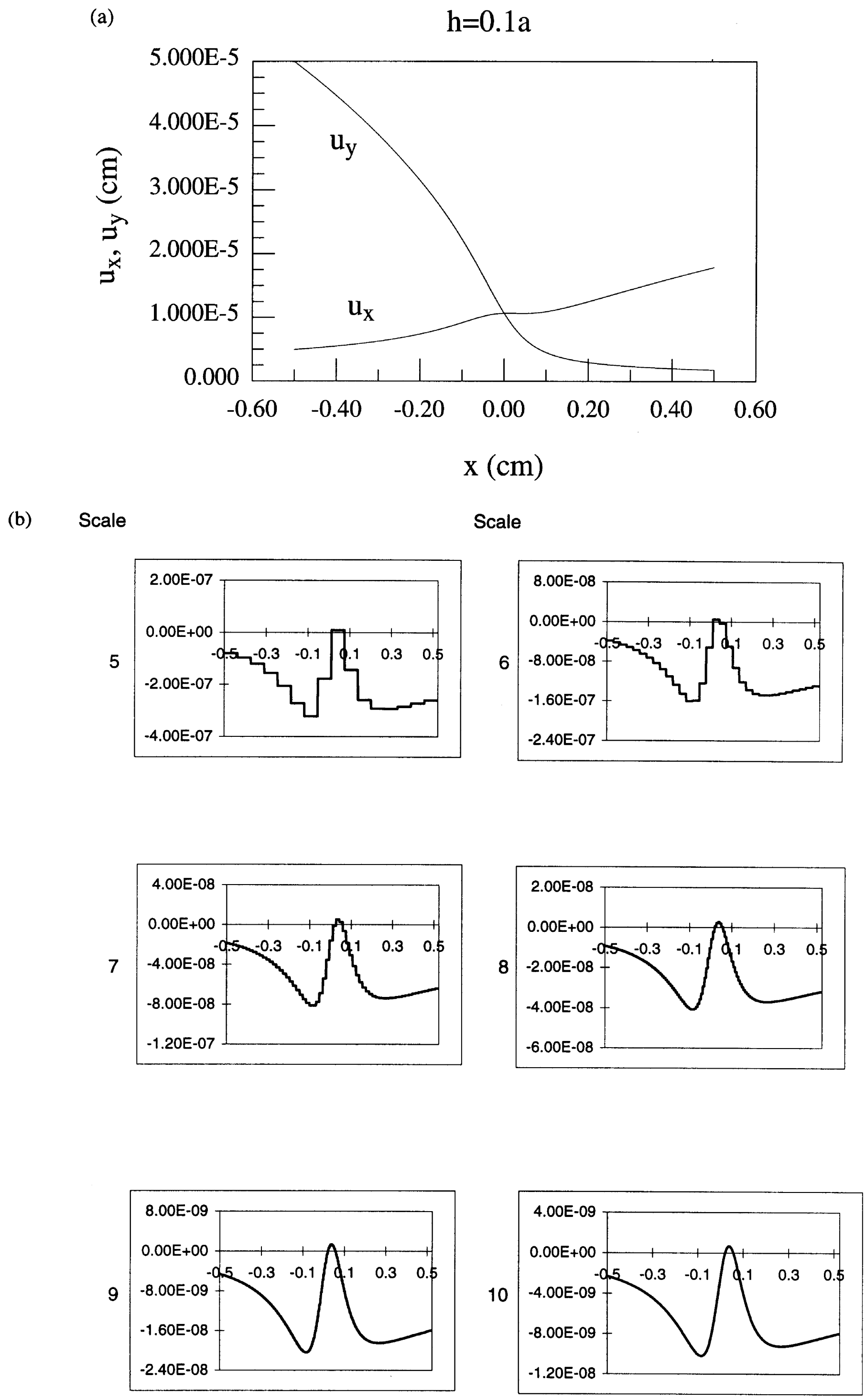


Fig. 12. (a) Displacement response along the horizontal line segment; (b) Haar wavelets of scales 5–10 for u_x; (c) Haar wavelets of scales 5–10 for u_y.

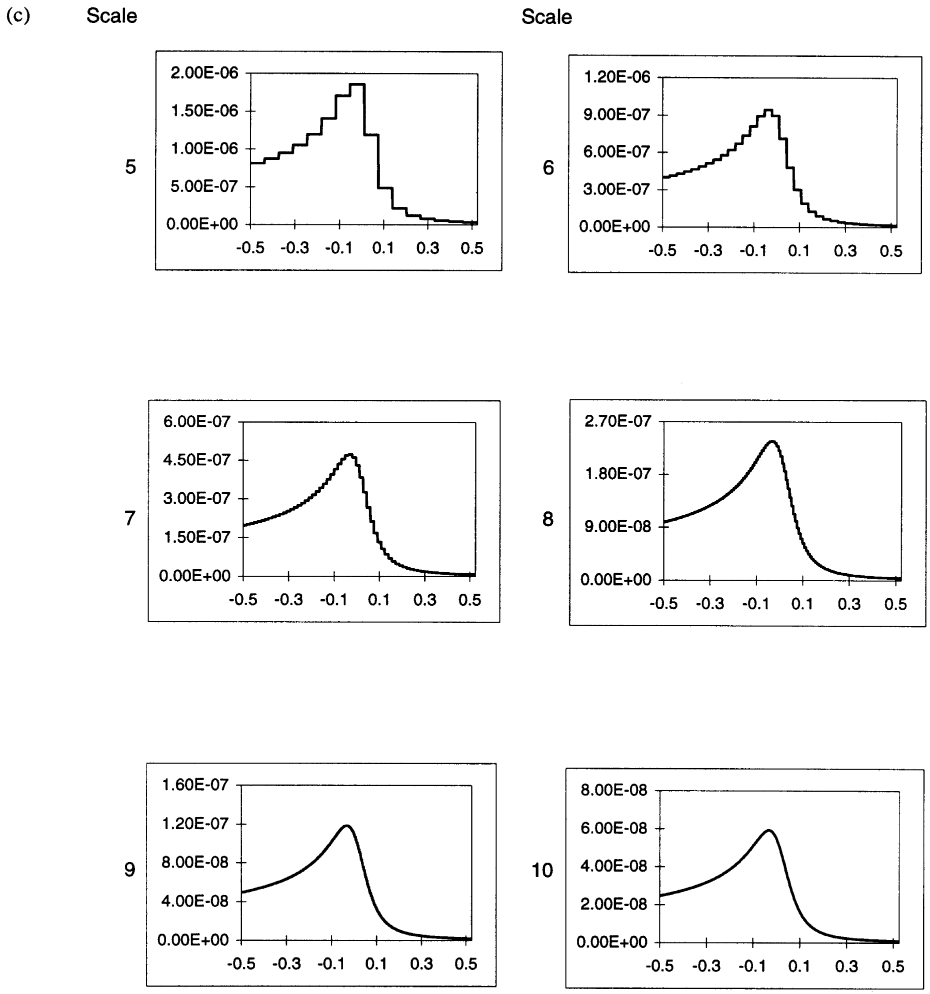


Fig. 12 continued.

response signals with the wavelet transform, provided that the response signals contain the local perturbation information caused by the damage near the damage sites.

3.5. Analyses with other wavelets

Because of their simplicity, the Haar wavelets have been used in the wavelet transforms in the preceding examples. Here we demonstrate that similar results can be obtained when other suitable wavelets are used to analyze the spatially distributed signals. To this end, we repeat the wavelet transforms for the deflection signals of the cracked beams with Gabor wavelets generated from the Gabor function in eqn (7). Shown in Fig. 13(a), (b) and (c) are the distributions of the Gabor wavelet coefficients of scale 7 for the deflection signals given in Figs 4, 6 and 8, respectively, where $|c_{j,k}|$ refers to the magnitude of the wavelet coefficients. Again we observe that peaks show up at

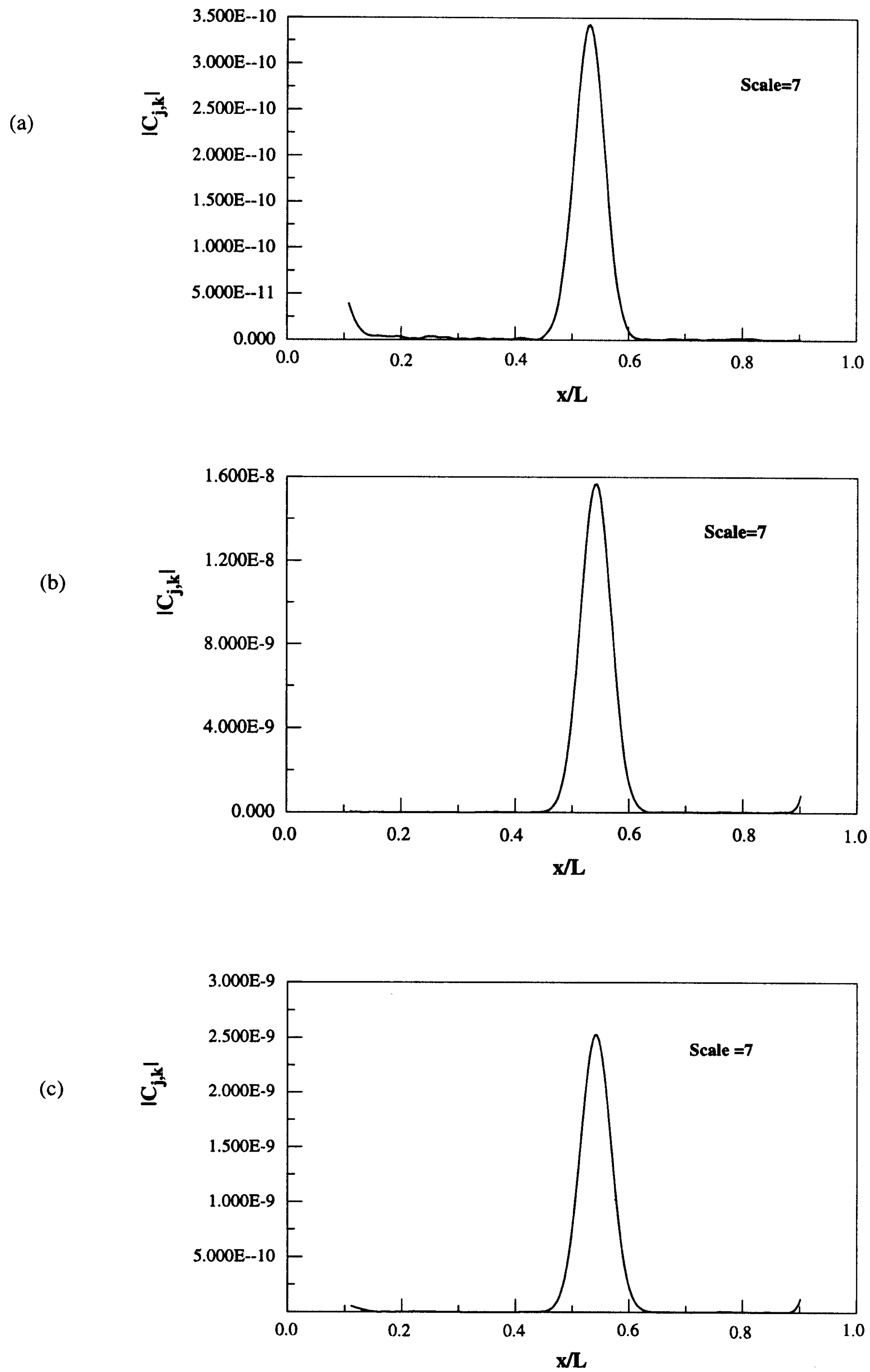


Fig. 13. The distributions of Gabor wavelet coefficients of scale 7 based on the following deflection responses of the cracked beam: (a) Fig. 4, (b) Fig. 6, (c) Fig. 8.

the crack location. In fact, the Gabor wavelets appear to capture well the local disturbance created by the existence of a crack. The reason for it is that the Gabor function has a small area of time-frequency window (smaller than those of many other functions, including the Haar function), which makes the Gabor wavelets well suited for wavelet analysis.

3.6. Sensitivity analysis

A key practical issue in developing and applying a damage detection method based on spatially distributed signals is the spatial resolution required in making the measurements. On the other hand, the achievable spatial signal resolution in practice depends on the particular sensor type and data collection method used in damage detection. This section provides insights for the sensitivity of the proposed wavelet transform method to the spatial resolution of sensor signals.

As a representative case, we study the earlier example of the simply supported beam shown in Fig. 3. In the previous wavelet transform (see Fig. 5) of the deflection signal (Fig. 4) for the cracked beam, no consideration was made of any practical limit of the signal's spatial resolution. In those analyses, we used 1024 uniformly distributed signal points for the half-meter beam, which were chosen for the convenience of evaluating the wavelet transforms. To seek the lower limit of the wavelet method for the beam problem in question, we have gradually reduced the number of uniformly distributed signal points from 1024 to 62 then to 31. During the ensuing wavelet transforms, any signal values needed at points other than the signal points were approximated from the values at the neighboring signal points using linear interpolation. The resulting Haar wavelet coefficients are shown in Fig. 14 (based on 62 signal points) and Fig. 15 (based on 31 signal points). These wavelet coefficient distributions indicate that the perturbation caused by the crack can still be detected at such relatively low spatial resolutions. However, when the number of signal points is reduced further to 15, no trace of the crack can be detected.

4. Summary and future work

Wavelet analysis is an alternative to Fourier analysis. The advantage of using wavelet analysis is that local features in a signal can be identified with a desired resolution. Based on this multiresolution property of the wavelet transform, it has been shown that a spatially distributed response signal can be analyzed with the wavelet transform and may be used for structural damage detection purposes, provided that the signal picks up perturbations induced by the presence of damage. Simulated deflection signals of a beam containing a transverse crack and the displacement response of a plate with a through-thickness crack have been used to demonstrate the feasibility of this damage detection method. It appears that damage can cause structural response perturbations near damage sites with inherent local features, and that these local features can be enhanced with the wavelet transform. A preliminary sensitivity analysis is also performed.

Further work is needed to validate and advance this damage detection method. For example, the sensitivity of this method to random signal noise must be further investigated, and actual test signals are needed to demonstrate the practicality of this method. It seems that the key to the success of this technique is the development of sensors that can be used to provide spatially distributed response signals that can pick up damage perturbation information. While these and

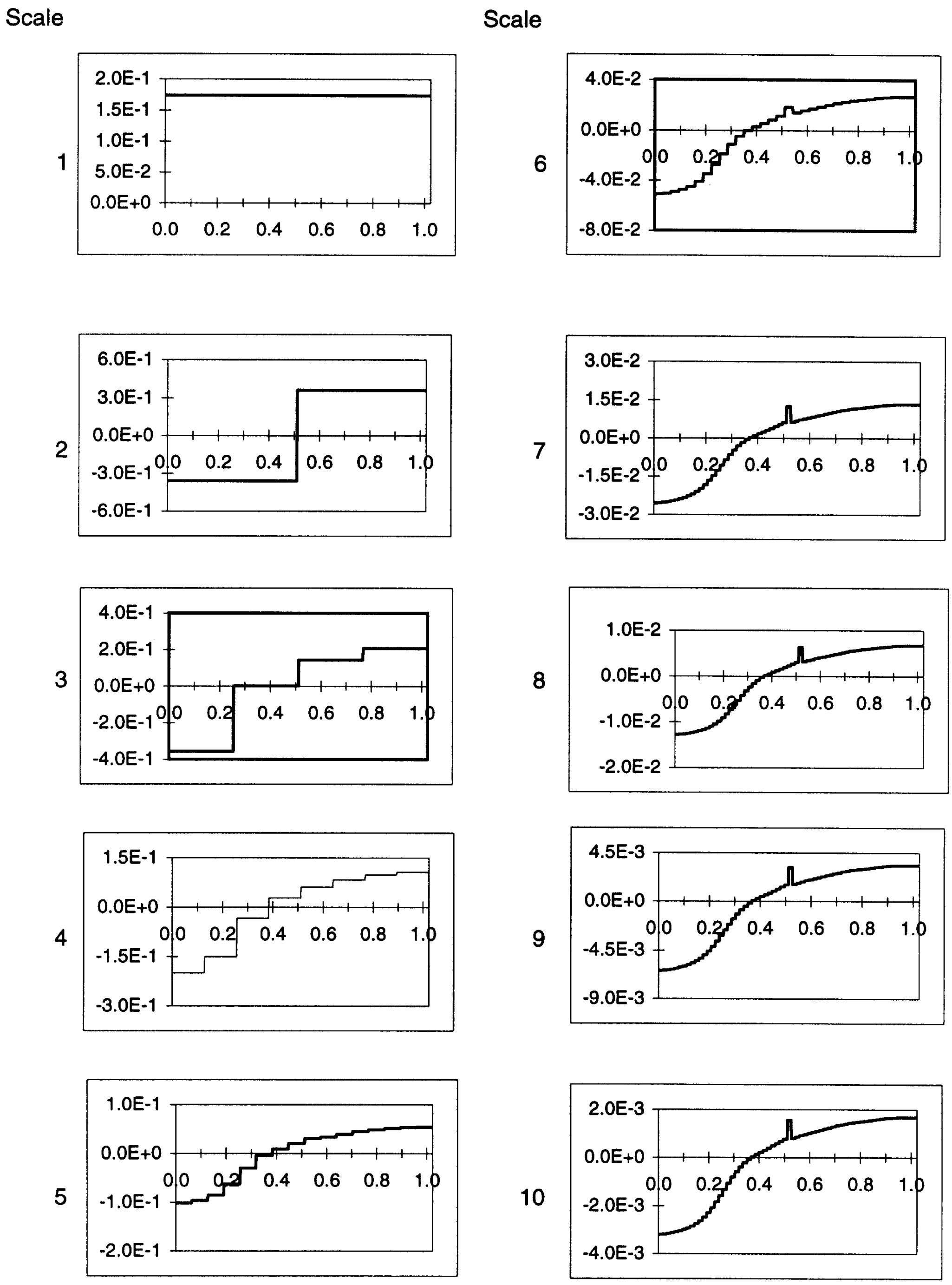


Fig. 14. The distributions of Haar wavelet coefficients for wavelets of scales 1–10 based on 62 sensor signals for the deflection response in Fig. 4.

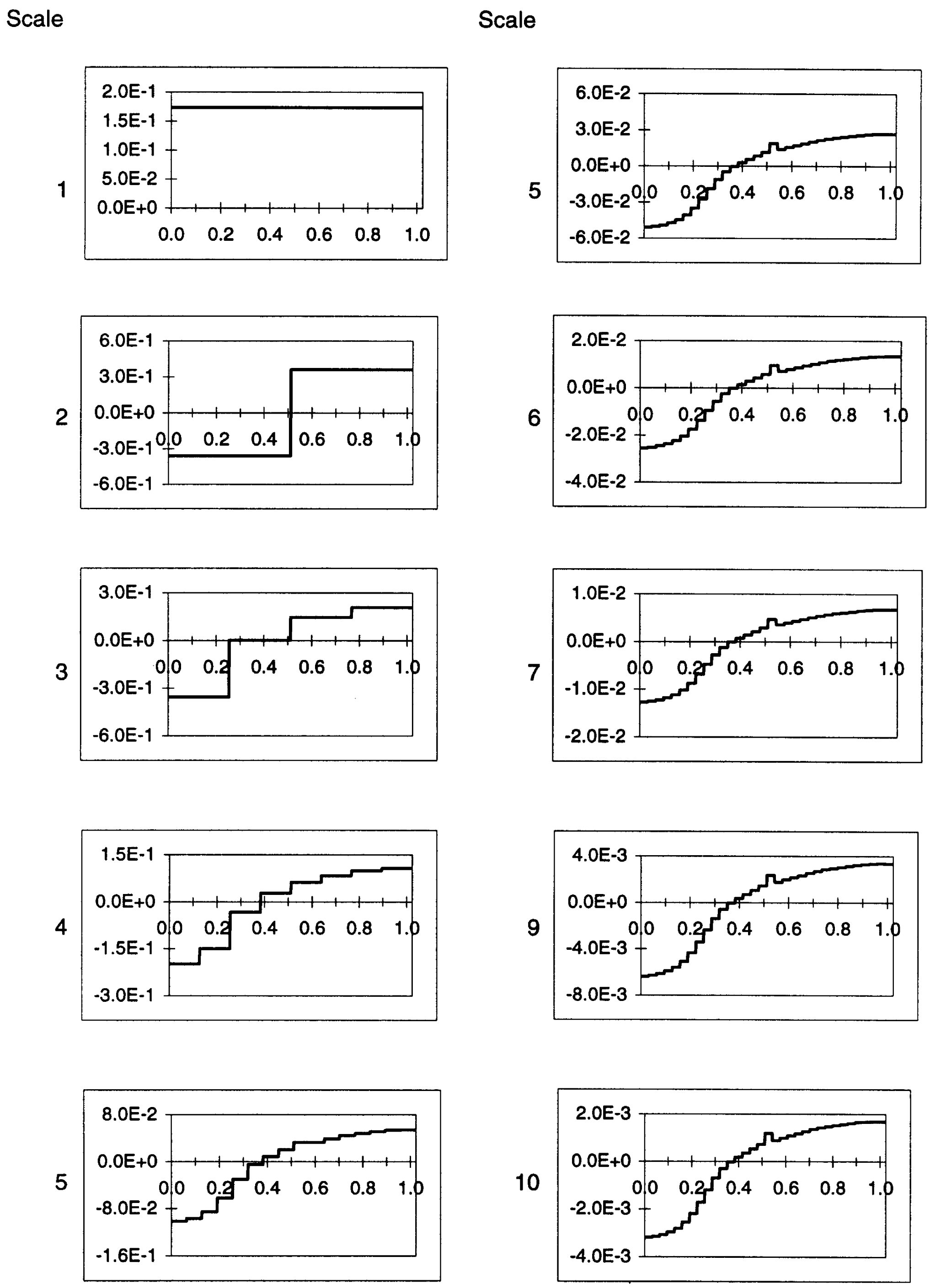


Fig. 15. The distributions of Haar wavelet coefficients for wavelets of scales 1–10 based on 31 sensor signals for the deflection response in Fig. 4.

other issues will be addressed in future communications by the current authors, it is hoped that this paper will provide both a stimulus and a basis for further studies by investigators in this area.

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